Note 1 (Systems of Linear Equations)

- Linear Function:
$$f(\alpha x_1 + \beta y_1, \alpha x_2 + \beta y_2, \dots, \alpha x_n + \beta y_n) \implies \alpha f(x_1, x_2, \dots, x_n) + \beta f(y_1, y_2, \dots, y_n)$$
- If $f : \mathbb{R}^n \to \mathbb{R}$ is linear, then
$$f(x_1, x_2, \dots, x_n) = c_1 x_1 + c_2 x_2 + \cdots + c_n x_n$$
- Affine function:
$$g(x_1, x_2, \dots, x_n) = f(x_1, x_2, \dots, x_n) + c_0$$
for a linear function $f : \mathbb{R}^n \to \mathbb{R}$ and constant $c_0 \in \mathbb{R}$.

Note 2 (Vectors/Matrices)

- Vectors $\vec{x} \in \mathbb{R}^n$
  - each x_i = component/element
  - size = # of elements (n)
  - vectors are equal if same size and elements are equal
- Standard unit vector: all elements 0 except one 1
- Vector multiplication:
  - $\vec{y}^T \vec{x}$ = dot product; $\sum x_i y_i$
  - $\vec{x}\vec{y}^T$ = matrix: $\begin{bmatrix} x_1 y_1 & x_1 y_2 & \cdots \\ x_2 y_1 & \ddots & \vdots \\ \vdots & \cdots & x_n y_n \end{bmatrix}$
- Matrix multiplication:
  - $\mathbf{AB}$: for each row of $\mathbf{A}$, multiply and sum for each col of $\mathbf{B}$
  - Associative, not commutative:
    $(\mathbf{AB})\mathbf{C} = \mathbf{A}(\mathbf{BC})$; $\mathbf{AB} \neq \mathbf{BA}$
- Identity matrix: $\mathbf{I}$; 1's along diagonal, 0's everywhere else

Note 3 (Linear Independence/Span)

- Set of vectors $\{\vec{v}_1, \vec{v}_2, \dots, \vec{v}_n\}$ is linearly dependent (LD) if $\alpha_1 \vec{v}_1 + \alpha_2 \vec{v}_2 + \cdots + \alpha_n \vec{v}_n = \vec{0}$, where some $\alpha_i \neq 0$
- Or, LD if v_i can be written as $\sum \alpha_j v_j$, where some $\alpha_j \neq 0$
- Linearly independent (LI) if $\sum \alpha_i \vec{v}_i = \vec{0}$ only if all $\alpha_i = 0$
- Span: set of all linear combinations of the vectors

Note 5 (Water Pumps)

- Transition matrix (flow from cols to rows):
$$\begin{array}{c c} & \begin{array}{ccc} A & B & C \end{array} \\ \begin{array}{c} A \\ B \\ C \end{array} & \begin{bmatrix} A \to A & B \to A & C \to A \\ A \to B & B \to B & C \to B \\ A \to C & B \to C & C \to C \end{bmatrix} \end{array}$$
- Columns sum to 1 $\implies$ conservative system (everything goes somewhere)

Note 6 (Matrix Inversion)

- $\mathbf{A}$ is invertible if there exists a $\mathbf{B}$ s.t. $\mathbf{AB} = \mathbf{BA} = \mathbf{I}$
- Finding inverse: Gaussian elimination, like solving $\mathbf{AX} = \mathbf{I}$; i.e. reduce $[\mathbf{A} \mid \mathbf{I}] \to [\mathbf{I} \mid \mathbf{A}^{-1}]$
- If invertible, then: (baby version of IMT)
  - rows and cols LI
  - $\mathbf{A}\vec{x} = \vec{b}$ has a unique solution for all $\vec{b}$
  - has a trivial nullspace
  - $\det \mathbf{A} \neq 0$

Note 7 (Vector Spaces)

- V is a vector space if:
  - $\vec{0} \in V$
  - $\vec{x}, \vec{y} \in V$ implies $\alpha \vec{x} + \beta \vec{y} \in V$ for all $\alpha, \beta \in \mathbb{R}$; i.e.
    - closed under vector addition (if $\vec{x}, \vec{y} \in V$ then $\vec{x} + \vec{y} \in V$)
    - closed under scalar multiplication (if $\vec{x} \in V$ then $\alpha \vec{x} \in V$ for all $\alpha \in \mathbb{R}$)
  - (among other things, but these are the most important)
- Basis of a vector space:
  - LI vectors, can express any $\vec{v} \in V$ as a linear combination of basis vectors, and is a minimum set of vectors that does so (implied by being LI)
- Dimension of a vector space = # of basis vectors
- All equivalent bases of a vector space must have the same dimension

Note 8 (Matrix subspaces)

- U is a subspace if it satisfies the 3 points in Note 7 (a subspace is a subset of a vector space)
- Column space: range($\mathbf{A}$) = span($\mathbf{A}$) = C($\mathbf{A}$) = span of cols of $\mathbf{A}$
- rank($\mathbf{A}$) = dim(span($\mathbf{A}$))
- Nullspace: Null($\mathbf{A}$) = N($\mathbf{A}$) = set of all $\vec{x}$ s.t. $\mathbf{A}\vec{x} = \vec{0}$
- nullity($\mathbf{A}$) = dim(N($\mathbf{A}$))
- rank–nullity theorem: rank($\mathbf{A}$) + nullity($\mathbf{A}$) = # of columns of $\mathbf{A}$

Note 9 (Eigenvalues/Eigenvectors)

- If $\mathbf{A}\vec{x} = \lambda \vec{x}$, then $\vec{x}$ is an eigenvector, λ is an eigenvalue of $\mathbf{A}$
- Calculating eigenvalues: solve $\det(\mathbf{A} - \lambda \mathbf{I}) = 0$ for λ
- Calculating eigenvectors: solve $(\mathbf{A} - \lambda \mathbf{I})\vec{v} = \vec{0}$ for $\vec{v}$
- Repeated eigenvalues: multiple eigenvectors, same eigenvalue; forms an eigenspace
- If $(\lambda_1, \vec{v}_1)$, $(\lambda_2, \vec{v}_2)$ are two distinct eigenpairs, then $\vec{v}_1, \vec{v}_2$ are LI.
- Characteristic polynomial: $\det(\mathbf{A} - \lambda I)$
- Steady states (water pumps): $\vec{x}$ s.t. $\mathbf{A}\vec{x} = \vec{x}$ (i.e. eigenspace for $\lambda = 1$)
- $\lim_{n \to \infty} \mathbf{A}^n \vec{x} = \lim_{n \to \infty} \lambda^n \vec{x}$ if $(\lambda, \vec{x})$ is an eigenpair of $\mathbf{A}$

Note 10 (Change of Basis/Diagonalization)

- If $\mathbf{T}\vec{u} = \vec{v}$, and $\vec{u}_A$ and $\vec{v}_B$ are vectors in the $\mathbf{A}$ and $\mathbf{B}$ bases respectively (i.e. columns of $\mathbf{A}$ and $\mathbf{B}$ are basis vectors in the new coordinate system), then arrows represent consecutive left-multiplication:
$$\begin{tikzcd} \vec{u} \arrow[r, "\mathbf{T}"] \arrow[d, bend right, "\mathbf{A}^{-1}"'] & \vec{v} \arrow[d, bend left, "\mathbf{B}^{-1}"] \\ \vec{u}_A \arrow[u, bend right, "\mathbf{A}"'] \arrow[r, "\mathbf{B}^{-1}\mathbf{TA}"'] & \vec{v}_B \arrow[u, bend left, "\mathbf{B}"] \end{tikzcd}$$
- Diagonalization: $\mathbf{A} = \mathbf{PDP}^{-1}$; $\mathbf{P}$ = matrix of eigenvectors, $\mathbf{D}$ = diagonal matrix of eigenvalues such that $\mathbf{A}\vec{v}_i = \lambda_i \vec{v}_i$
$$\mathbf{P} = \begin{bmatrix} | & & | \\ \vec{v}_1 & \cdots & \vec{v}_n \\ | & & | \end{bmatrix} \qquad \mathbf{D} = \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & \lambda_n \end{bmatrix}$$
- Only diagonalizable if eigenvalues are linearly independent (i.e. if all eigenvalues are distinct)

Other

- $(\mathbf{AB})^T = \mathbf{B}^T \mathbf{A}^T$
- rotation matrix by θ counterclockwise: $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$
- $\det \begin{bmatrix} a & b \\ c & d \end{bmatrix} = ad - bc$
- $\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \dfrac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$
- $\det(\mathbf{AB}) = \det(\mathbf{A})\det(\mathbf{B})$
- $\det(\mathbf{A}^{-1}) = \det(\mathbf{A})^{-1}$
- $\det(\mathbf{A})$ = product along diagonal if $\mathbf{A}$ is triangular
  - eigenvalues of a triangular matrix are the values along diagonal

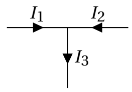
Note: Do not blindly copy down the content on this cheatsheet (this is one of the reasons why I typed this in LaTeX, as you cannot just print it out and use it). My goal in making my cheatsheet publicly available is to ease your studying and to provide a general guideline for the kinds of things that you could put on your cheatsheet. The most helpful part of a cheatsheet is the process of making it—you synthesize and review the course material yourself and pick out what *you* think is important.

I guarantee that there are concepts in the notes/lectures/discussions not on my cheatsheet that you should know, and I also guarantee that there are things on my cheatsheet that may be of no use to you. This is a collection of items that helped *me* when I took the class, and only you know what benefits you the most. My advice is to make your own cheatsheet first, without referencing mine, and then going over my cheatsheet afterward to add things that you may have missed.

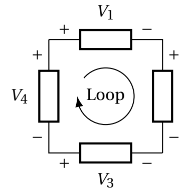
Good luck on your exams!

Note 11 (Circuits)

- Ohm's law: $V = IR$
- KCL: $I_{in} = I_{out}$
- KVL: $\sum_{\text{loop}} V_k = 0$; $- \to + =$ add, $+ \to - =$ subtract

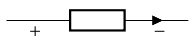


$I_1 + I_2 = I_3$



$-V_1 - V_2 + V_3 + V_4 = 0$

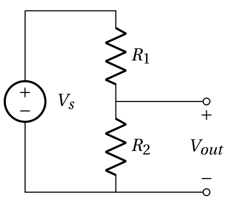
- NVA:
  - Label everything
  - Passive sign convention: current goes into +, out of −



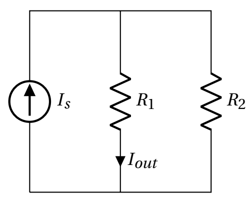
  - Write KCL at each unknown node
  - Substitute Ohm's law for each current
  - Solve for desired values

Note 12 (Resistive Touchscreen)

- Voltage divider:
- Current divider:

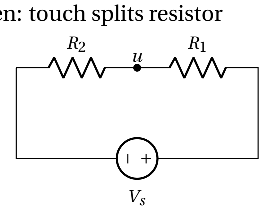


$V_{out} = V_s \frac{R_2}{R_1+R_2}$



$I_{out} = I_s \frac{R_2}{R_1+R_2}$

- $R = \rho \frac{L}{A} = \rho \frac{\text{length}}{\text{area}}$, where ρ = resistivity
- Resistive touchscreen: touch splits resistor



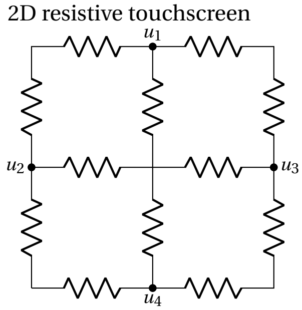
$$R_2 = \rho \frac{L_{touch}}{A} \qquad R_1 = \rho \frac{L - L_{touch}}{A} \qquad u = V_s \frac{L_{touch}}{L}$$

Note 13 (Power)

- Power: $P = VI = \frac{V^2}{R} = I^2 R$
- Voltmeter: connected *in parallel* to element to measure voltage drop
- Ammeter: embedded in the circuit *in series* to measure current

Note 14 (2D Resistive Touchscreen)

- 2D resistive touchscreen



- Powering $u_1 \to u_4$: measure y position

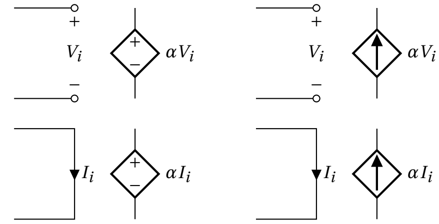
$$V_{out} = V_s \frac{L_{touch,vertical}}{L}$$

- Powering $u_2 \to u_3$: measure x position

$$V_{out} = V_s \frac{L_{touch,horizontal}}{L}$$

Note 15 (Superposition, Equivalences)

- Dependent sources:



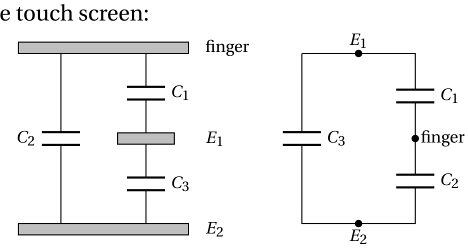
- Superposition:
  - for each independent source:
    - replace voltage source with wire, current source with open circuit
    - leave everything alone, find value (keep the same signs!)
  - sum up everything
- Resistor equivalences:
  - Parallel: $R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$
  - Series: $R_{eq} = R_1 + R_2$
- Voltage drop is equal through parallel branches (adjacent to same nodes)
- Current is equal through elements in series (by KCL)

Note 16 (Capacitors)

- Capacitors:
  - charge (on positive plate) = $Q = CV$; C = capacitance
  - $I = C \frac{dV}{dt}$
  - if constant current, then $I = C \frac{\Delta V}{\Delta t}$ and $It = C(V(t) - V(0))$
- Capacitor equivalences:
  - Parallel: $C_{eq} = C_1 + C_2$
  - Series: $C_{eq} = \frac{C_1 C_2}{C_1 + C_2}$
- $C = \varepsilon \frac{A}{d} = \varepsilon \frac{\text{area}}{\text{distance}}$, where ε = permitivity
- Energy: $E = \frac{1}{2} C V^2$

Note 17 (Capacitive Touchscreen)

- Capacitive touch screen:



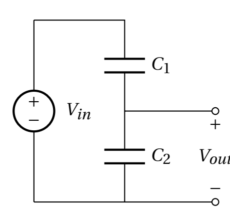
- Touch adds parallel capacitors (C_1, C_2) $\implies$ increased capacitance

Note 17B (Charge Sharing)

- Charge sharing steps:
  - Draw/label phases, keep polarity/signs for elements consistent through phases
  - For all floating nodes in phase 2, use charge conservation; find total charge on adjacent plates (keep + and − plates in mind!)
  - Equate with the total charge on the *same* plates in phase 1

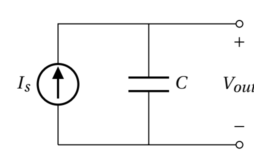
Other

- Capacitive divider:



$V_{out} = V_{in} \frac{C_1}{C_1 + C_2}$

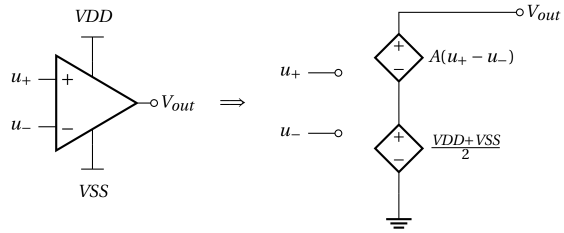
- Charging a capacitor:



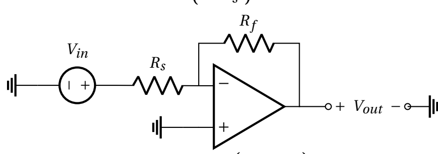
$V_{out} = \frac{I_s}{C} t$

Note 18/19 (Op Amps)

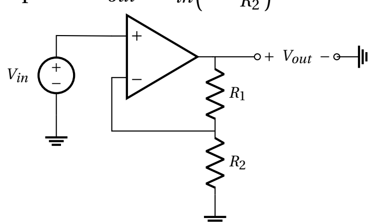
- Op amp:



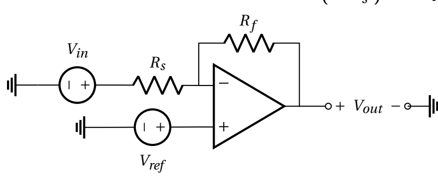
- Ideal op amp:
  - $A \to \infty$
  - No current through u_+, u_-
  - $u_+ - u_- = 0$
- Inverting Amplifier: $V_{out} = V_{in}\left(-\frac{R_f}{R_s}\right)$



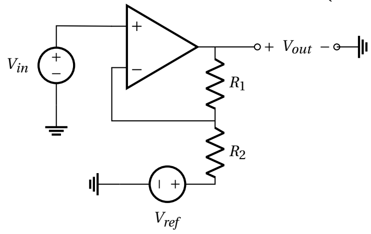
- Noninverting Amplifier: $V_{out} = V_{in}\left(1 + \frac{R_1}{R_2}\right)$



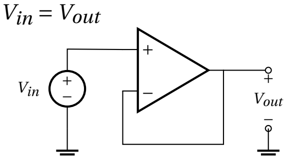
- Inverting Amplifier w/ reference: $V_{out} = V_{in}\left(-\frac{R_f}{R_s}\right) + V_{ref}\left(1 + \frac{R_f}{R_s}\right)$



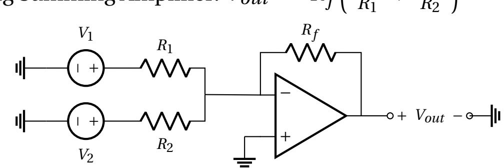
- Noninverting Amplifier w/ reference: $V_{out} = V_{in}\left(1 + \frac{R_1}{R_2}\right) - V_{ref}\left(\frac{R_1}{R_2}\right)$



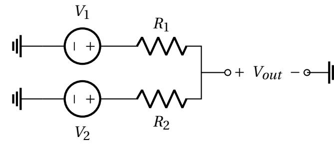
- Unity Gain Buffer: $V_{in} = V_{out}$



- Inverting Summing Amplifier: $V_{out} = -R_f\left(\frac{V_{in1}}{R_1} + \frac{V_{in2}}{R_2}\right)$



- Voltage Summer: $V_{out} = V_1\left(\frac{R_2}{R_1+R_2}\right) + V_2\left(\frac{R_1}{R_1+R_2}\right)$



Note 21 (Inner Products)

- (Euclidean) Inner product: $\langle \vec{x}, \vec{y} \rangle = \vec{x} \cdot \vec{y} = \vec{x}^T \vec{y} = \sum x_i y_i$
- $\vec{x}$, $\vec{y}$ are orthogonal if $\langle \vec{x}, \vec{y} \rangle = 0$
- $\langle a\vec{x}, \vec{y} \rangle = a\langle \vec{x}, \vec{y} \rangle$ and $\langle \vec{x} + \vec{y}, \vec{z} \rangle = \langle \vec{x}, \vec{z} \rangle + \langle \vec{y}, \vec{z} \rangle$
- Norm: $\|\vec{x}\| = \sqrt{\langle \vec{x}, \vec{x} \rangle}$ = length/magnitude of vector
- Alternate definition: $\langle \vec{x}, \vec{y} \rangle = \|\vec{x}\| \|\vec{y}\| \cos\theta$
- Cauchy-Schwarz inequality: $|\langle \vec{x}, \vec{y} \rangle| \le \|\vec{x}\| \|\vec{y}\|$

Note 22 (Correlation)

- Cross-correlation: $\text{corr}_{\vec{x}}(\vec{y})[k] = \sum_{i=-\infty}^{\infty} x[i]\, y[i-k]$
- $\vec{x}[i], \vec{y}[i] = 0$ outside of defined range
- $\text{corr}_{\vec{x}}(\vec{y})[k] = \text{corr}_{\vec{y}}(\vec{x})[-k]$; they're mirrored
- Autocorrelation: $\text{corr}_{\vec{x}}(\vec{x})$
- Circular correlation:

$$\text{circcorr}(\vec{x}, \vec{y}) = \begin{bmatrix} \text{——} & \text{rows are all} & \text{——} \\ \text{——} & \text{circular shifts} & \text{——} \\ \text{——} & \text{of } \vec{y} & \text{——} \end{bmatrix} \vec{x}$$

Note 23 (Projection/Least Squares)

- Projection of $\vec{b}$ onto $\vec{a}$: $\text{proj}_{\vec{a}}(\vec{b}) = \frac{\langle \vec{b}, \vec{a} \rangle}{\langle \vec{a}, \vec{a} \rangle} \vec{a}$
- Scalar projection of $\vec{b}$ onto $\vec{a}$: $\frac{\langle \vec{b}, \vec{a} \rangle}{\langle \vec{a}, \vec{a} \rangle}$
- Projection onto subspace: if columns of $\mathbf{A}$ are orthogonal, then $\text{proj}_{\mathbf{A}}(\vec{b}) = \sum \text{proj}_{\vec{a}_i}(\vec{b})$ where $\vec{a}_i$ are columns of $\mathbf{A}$; if not, use least squares
- Least squares: to minimize the error $e = \left\| \mathbf{A}\vec{x} - \vec{b} \right\|$, we have $\hat{\vec{x}} = \left(\mathbf{A}^T\mathbf{A}\right)^{-1}\mathbf{A}^T\vec{b}$
- Setting up least squares:
  - $\mathbf{A}$ = matrix of known values/coefficients
  - $\vec{x}$ = vector of variables
  - $\vec{b}$ = vector of constants
- $\mathbf{A}^T\mathbf{A}$ is invertible if $\mathbf{A}$ has LI columns (i.e. can only apply least squares if $\mathbf{A}$ has LI columns)

Other

- Trilateration:
  - n variables $\Longrightarrow$ n equations if linear, $n+1$ equations if nonlinear (subtract from one equation to linearize)
  - in space: n dimensions $\Longrightarrow$ $n+1$ equations for circles/spheres; one is sacrificed to linearize
  - if delays are unknown, need $n+2$ equations; sacrifice one for reference, sacrifice another to linearize
- Units (good to double check calculations)
  - Current: A = C/s = charge/time
  - Voltage: V = J/C = energy/charge
  - Resistance: Ω = V/A
  - Power: W = J/s = energy/time
  - Capacitance: F = C/V = charge/volt